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# Al – Structure On F – Relation L - Fuzzy Supratopological Systems

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**Abstract:**

*We introduced, Algebraic structure on L-fuzzy topological system which is described in L-fuzzy relation. Moreover, the paper Algebraic structure on F-relation L-fuzzy supratopological system provide with cloudless examples.*
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**Keywords:** BCK/BCI Algebra,TM-Algebra, Fuzzy Relations, Fuzzy Topology

## 1.INTRODUCTION

Fuzzy concept introduced by L.A.Zadeh [25] at 1965 and developed the Fuzzy relation at 1971. Chang [18], Wong [24] ,Lowen [22] and others developed the fuzzy topological spaces. In 2010, Tamilarasi and Manimegalai introduced a new class of algebras called TM-algebras [23].

We introduced the concept in [1], Fuzzy Topological subsystem on a TM-algebra. We studied in [2], L− Fuzzy Topological TM-system. We developed the concept in [3], L− Fuzzy Topological TM-subsystem. In [4], [5] we studied Fuzzy SupratopologicalTMsystem, Fuzzy α− supracontinuous functions. In this paper, discuss the notion of an AL – Structure on F – Relation L – Fuzzy Supratopological systems and investigate some simple properties.

## 2. PRELIMINARIES

In this section we recall some basic definitions that are required in the sequel.

**Definition 2.1.**

For any non-empty set X, $\mu : X \to [0, 1]$ is called a fuzzy set of X.

**Definition 2.2.**

The union $A \cup B$, of two fuzzy sets A and B of a set X, is defined to be the fuzzy set $(A \cup B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}\ \forall\, x \in X$.

**Definition 2.3.**

The intersection $A \cap B$, of two fuzzy sets A and B of a set X, is defined to be the fuzzy set $(A \cap B)(x) = \text{Min}\{\mu_A(x), \mu_B(x)\}\ \forall\, x \in X$.

**Definition 2.4.**

For any two fuzzy sets A and B of X . $A \subset B$ if $A(x) \le B(x)\ \forall\, x \in X$.

**Definition 2.5.**

Let A be a fuzzy set of X. Then the complement of A denoted by, $A_0$ , is defined to be $A_0(x) = 1 - A(x)\ \forall\, x \in X$.

**Definition 2.6.**

A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions.

(1) φ, X ∈ T
(2) If A, B ∈ T then A ∩ B ∈ T
(3) If $A_i$∈ T for each i∈ I then ∪I$A_i$∈ T where I is an indexing set.

**Definition 2.7.**

A TM-Algebra ( X, ∗, 0 ) is a non-empty set X with a constant 0 and a binary operation ∗ satisfying the following axioms:

(1) x ∗ 0 = x
(2) (x ∗ y) ∗ (x ∗ z) = z ∗ y for all x, y, z ∈ X.

**Definition 2.8.** Fuzzy TM-Subalgebra

A fuzzy subset μ of a TM-Algebra ( X, ∗, 0 ) is called a fuzzy TM-Subalgebra of X if , for all x, y ∈ X, μ(x ∗ y) ≥ min { μ(x) , μ(y)}

**Definition 2.9.** μ and σ are two fuzzy sets in a fuzzy topological space (X, T). σ is said to be an interior of μ if μ is a neighbourhood of σ and μ ⊃ σ.

**Definition 2.10.** Fuzzy Relation

Consider the Cartesian product A × B = {(x, y) : x ∈ A, y ∈ B} where A and B in universal sets U and V correspondingly. A fuzzy relation on A × B denoted by R or R(x, y) is defined as the set R $= \{(\mathrm{x,y}),\ \mu_R(x,y): (\mathrm{x,y}) \in A \times B,\ \mu_R(x,y) \in [0,1]\}$

**Definition 2.11.** The union of fuzzy relations $R_1$ and $R_2$ is denoted by $R_1 \cup R_2$ is defined by $\mu_{R_1 \cup R_2}(x,y) = Min\,\{\mu_{R_1}(\mathrm{x,y}), \mu_{R_2}(\mathrm{x,y})\}, (\mathrm{x,y}) \in A \times B$

The intersection of fuzzy relations $R_1$ and $R_2$ is denoted by $R_1 \cap R_2$is defined by $\mu_{R_1 \cap R_2}(x,y) = Max\,\{\mu_{R_1}(\mathrm{x,y}), \mu_{R_2}(\mathrm{x,y})\}, (\mathrm{x,y}) \in A \times B$

## 3. AL - STRUCTURE ON F- RELATION L - FUZZY SUPRATOPOLOGICAL SYSTEMS

**Definition 3.1.**

X, Y are TM-Algebras. $R_1(x,y), R_2(x,y)$ are an L-fuzzy relations of X, Y. AL - structrue on F - relation L - Fuzzy Supratopological System is a family T of L - fuzzy subalgebras in (X, Y, T) which is satisfies the conditions :

i) φ, X ∈ T ii) If $\mu_i$(x, y) ∈ T for each i∈ I then $\cup_I \mu_i$(x, y) ∈ T where I is an indexing L -subalgebra.

**Example 3.2.** The set X = {0, 1, 2}, Y = {0, 1, 2} with the cayley table

| ∗ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 1 | 2 | 0 |
| 2 | 2 | 0 | 1 |

AnL- fuzzy relations $R_1(x,y), R_2(x,y)$ are on the sets X, Y is

| X, Y | 0 | 1 | 2 |
|---|---|---|---|
| 0 | (0, 0) | (0, 1) | (0, 2) |
| 1 | (1, 0) | (1, 1) | ( 1, 2) |

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| | | | |
|---|---|---|---|
| 2 | (2, 0) | (2, 1) | (2, 2) |

The F – Relations L – Subalgebras$\mu_i : X \to [0,1], i = 1,2,3, \vartheta_i : Y \to [0,1], i = 1,2,3$ are

$$\mu_1(x,y) = \begin{cases} t_4 \text{ if } x = (0,0) \\ t_3 \text{ if } x = (0,1) \\ t_1 \text{ if } x = (0,2) \end{cases} \mu_2(x,y) = \begin{cases} t_6 \text{ if } x = (1,0) \\ t_5 \text{ if } x = (1,1) \\ t_3 \text{ if } x = (1,2) \end{cases} .3\ \mu_3(x,y) = \begin{cases} t_6 \text{ if } x = (2,0) \\ t_3 \text{ if } x = (2,1) \\ t_1 \text{ if } x = (2,2) \end{cases}$$

$$\vartheta_1(x,y) = \begin{cases} t_8 \text{ if } x = (0,0) \\ t_6 \text{ if } x = (0,1) \\ t_2 \text{ if } x = (0,2) \end{cases} \vartheta_2(x,y) = \begin{cases} t_4 \text{ if } x = (1,0) \\ t_2 \text{ if } x = (1,1) \\ t_1 \text{ if } x = (1,2) \end{cases} \vartheta_3(x,y) = \begin{cases} t_5 \text{ if } x = (2,0) \\ t_4 \text{ if } x = (2,1) \\ t_2 \text{ if } x = (2,2) \end{cases}$$

A family $T = \{\mu_1, \mu_2, \mu_{3,} \vartheta_1, \vartheta_2, \vartheta_3\}$ which is satisfying the AL – Structure on F - relation L-fuzzy Supratopolalogical system (X,Y, T).

**Definition 3.3.**

X, Y are TM-Algebras. R$_1(x,y)$ and R$_2(x,y)$are an L-fuzzy relations of X, Y. AL - structrue on L-Fuzzy relation supratopological System (X, Y, T). L-Fuzzy relation subalgebra$\mathcal{N}$ in L-fuzzy supratopologicalsystem, is an L-fuzzy relation neighbourhood of an L-fuzzy relation subalgebra$\mathcal{M}$ if there exist an T-open L-fuzzy relation subalgebra$\mathcal{D}$ such that $\mathcal{M} \subset \mathcal{D} \subset \mathcal{N}$

ie$\mathcal{M}$(x, y) ≤ $\mathcal{D}$(x, y) ≤ $\mathcal{N}$ (x, y) for all x ∈ X, y ∈ Y

**Example 3.4.**

$\mu_i$(x, y) , i = 1, 2, 3 , $\vartheta_i$( x, y) , i = 1, 2, 3 are an L-fuzzy relation subalgebras of anL-fuzzy supratopologicalsystem given in example 3.2

T = { φ, X,$\mu_1, \mu_2, \mu_3, \vartheta_1, \vartheta_2, \vartheta_3$} is an L-fuzzy relation on L-fuzzy supratopologicalsystem (X, Y, T). $\mu_2$ (x, y) is L-fuzzy relation neighbourhood of anL-fuzzy relation subalgebra$\mu_1$(x, y) for $\mu_1$ (x, y) ≤ $\vartheta_2$(x, y) ≤ $\mu_2$ (x, y)

**Definition 3.5.**

X, Y are TM-Algebras. R$_1(x,y)$ and R$_2(x,y)$are an L-fuzzy relations of X, Y. AL - structrue on L-Fuzzy relation supratopological System (X, Y, T). The L-fuzzy relation interior of $\mu^*$ (x, y) is the union of all L-fuzzy relation open subalgebras contained in $\mu^*$ (x, y) and it is denoted by $(\mu^*)^\circ$(x, y). That is $(\mu^*)^\circ$ (x, y) = ∪ {μ(x, y) : μ(x, y) ⊆$\mu^*$ (x, y), μ(x, y) ∈ (X, Y, T)

**Example 3.6.**

$\mu_i$(x, y) , i = 1, 2, 3 , $\vartheta_i$( x, y) , i = 1, 2, 3 are an L-fuzzy relation subalgebras of the L-fuzzy supratopologicalsystem given in example 3.2

T = { φ, X,$\mu_1, \mu_2, \mu_3, \vartheta_1, \vartheta_2, \vartheta_3$} is L-fuzzy relation on fuzzy supratopologicalsystem (X, Y, T).

The L-fuzzy relation interior of $\mu^*$ (x, y) = $\mu_3(x,y)$ is$(\mu_3)^\circ(x,y)$ = ∪ {$\mu_1, \vartheta_2, \vartheta_3$} = $\mu_1$(x, y)

**Theorem 3.7.**

X, Y are TM-Algebras. R$_1(x,y)$ and R$_2(x,y)$are an L-fuzzy relations of X, Y. AL - structrue on L-Fuzzy relation supratopological System (X, Y, T). L-Fuzzy relation subalgebra$\mathcal{A}$(x, y) is open in (X, Y, T) if and only if for each L-fuzzy relation subalgebra$\mathcal{B}$(x, y) contained in $\mathcal{A}$(x, y), $\mathcal{A}$ (x, y) is L-fuzzy relation neighbourhood of $\mathcal{B}$ (x, y).

**Proof:**

An L-fuzzy relation subalgebra$\mathcal{A}$ (x, y) is open in (X, Y, T).

$\mathcal{B}$ (x, y) is any L-fuzzy relation subalgebra contained in$\mathcal{A}$ (x, y). Since $\mathcal{A}$ (x, y) is open and $\mathcal{B}$ (x, y) ⊂$\mathcal{A}$(x, y), $\mathcal{B}$ (x, y) ⊂$\mathcal{A}$ (x, y) ⊂$\mathcal{A}$ (x, y)

∴$\mathcal{A}$ (x, y) is L-fuzzy relation neighbourhood of $\mathcal{B}$ (x, y).

Conversely, for each L-fuzzy relation subalgebra$\mathcal{B}$ (x, y) contained in $\mathcal{A}$(x, y), $\mathcal{A}$ (x, y) is L-fuzzy neighbourhood of $\mathcal{B}$ (x, y).

for $\mathcal{A}$ (x, y) ⊂$\mathcal{A}$(x, y) , by our assumption, $\mathcal{A}$ (x, y) is anL-fuzzy relation neighbourhood of $\mathcal{A}$ (x, y).

Hence there exits an open L-fuzzy relation subalgebra$\mathcal{O}$ (x, y) such that $\mathcal{A}$ (x, y) ⊂$\mathcal{O}$ (x, y) ⊂$\mathcal{A}$ (x, y).

Hence $\mathcal{A}$(x, y) = $\mathcal{O}$ (x, y) and $\mathcal{A}$ (x, y) is open in (X, Y, T)

**Definition 3.8.**

X, Y are TM-Algebras. $R_1(x,y)$ and $R_2(x,y)$are an L-fuzzy relations of X, Y. AL - structrue on L-Fuzzy relation supratopological System (X, Y, T). μ(x, y) is L-fuzzy relation subalgebra in (X, Y, T). The collection of L-fuzzy relation neighbourhood of μ(x, y) is the set $\mathcal{U}$(x, y) is said to be anL-fuzzy relation neighbourhood system of μ(x, y).

**Theorem 3.9.**

X, Y are TM-Algebras. $R_1(x,y)$ and $R_2(x,y)$are anL-fuzzy relations of X, Y. AL - structrue on L-Fuzzy relation supratopological System (X, Y, T). μ(x, y) is an L-fuzzy relation subalgebra in (X, Y, T). $\mathcal{U}$ (x, y) is anL-fuzzy relation neighbourhood system of L-fuzzy relation subalgebraμ(x, y). then

i)The finite intersections of elements of $\mathcal{U}$(x, y) belong to $\mathcal{U}$ (x, y)

ii)An L-Fuzzy relation subalgebra of (X, Y, T) which contain a element of $\mathcal{U}$ (x, y) belong to $\mathcal{U}$ (x, y)

**Proof:**

i) AnL-fuzzy relations $R_1(x,y)$ and $R_2(x,y)$are anL-fuzzy supratopological system(X, Y, T). μ(x, y) is an L-fuzzy relation subalgebra in (X, Y, T).

$\mathcal{U}$ (x, y) is an L-fuzzy relation neighbourhood system of μ(x, y).

The elements g(x, y), h(x, y) ∈$\mathcal{U}$ (x, y) Hence g(x, y) and h(x, y) are an L-fuzzy relation neighbourhood of μ(x, y).

Thus there exits open an L-fuzzy relation subalgebras$g_\circ$(x, y) and $h_\circ$ (x, y) Such that μ(x, y) ⊂$g_\circ$ (x, y) ⊂ g(x, y) and μ(x, y) ⊂$h_\circ$ (x, y) ⊂ h(x, y) respectively.

Hence, μ(x, y) ⊂$g_\circ$(x, y) ∩ $h_\circ$ (x, y) ⊂ g(x, y) ∩ h(x, y)

⇒g(x, y) ∩ h(x, y) is an L-fuzzy relation neighbourhood of μ(x, y).

Hence, the intersection of two elements of$\mathcal{U}$ (x, y) is again a element of $\mathcal{U}$ (x, y).

Hence the intersection of any finite number of elements of $\mathcal{U}$ (x, y) is again a element of $\mathcal{U}$ (x, y)

ii) g(x, y) is an L-fuzzy relation subalgebra that contains a element of $\mathcal{U}$ (x, y) say u(x, y).

Hence, g(x, y) contains a neighbourhood u(x, y) of μ(x, y).
That is u(x, y) ⊂ g(x, y) , u(x, y) ∈$\mathcal{U}$ (x, y)
since u(x, y) is anL-fuzzy relation neighbourhood of μ(x, y) then by definition there exists a open L-fuzzy relation subalgebra o(x, y).
⇒μ(x, y) ⊂ o(x, y) ⊂ u(x, y) ⊂ g(x, y).
Therefore μ(x, y) ⊂ o(x, y) ⊂ g(x, y)
⇒g(x, y) is an L-fuzzy relation neighbourhood of μ(x, y).
∴ g(x, y) ∈$\mathcal{U}$ (x, y)

## References

[1] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy Topological subsystem on a TM-algebra, *Intetnational Journal of Pure and Applied Mathematics*,Vol.94,No.3 2014, 439-449.
[2] Annalakshmi. M., Chandramouleeswaran. M, On L− Fuzzy Topological TM-system, Intetnational *Journal of Mathematical Science and Engineering Applications*, Vol.8,No.IV (July 2014) PP. 135 -145.
[3] Annalakshmi. M., Chandramouleeswaran. M, On L− Fuzzy Topological TM- subsystem, *Italian Journal of Pure and Applied Mathematics* - N. 33 - 2014 (359-368) 8.
[4] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy supratopological TM-system, *International Journal of Pure and Applied Mathematics*,Vol.98,No.5 2015, 55-61.
[5] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy and L− Fuzzy compactness in a Fuzzy Topological TM-system, *Global Journal of Pure and Applied Mathematics*,Vol11, No.1 2015, PP491-498.
[6] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy α− supracontiuous functions, *International Journal of Mathematical Archive* -6(5),2015,116-119.
[7] Annalakshmi. M., Chandramouleeswaran. M, Interior and Closure of Fuzzy open sets in a Fuzzy Topological TM-system, *Global Journal of Pure and Applied Mathematics*,Vol.11,No.5 (2015), PP 3157-3164.
[8] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy Connected on Fuzzy Topological TM-system, *International Journal of Pure and Applied Mathematics*- Vol-112, No.5, pp.93-102.
[9] Annalakshmi. M., Chandramouleeswaran. M, Fuzzy Supratopological TM-Subsystem, *Mathematical Sciences International Research Journal*, Vol 6, Spl Issue Dec 2017 , PP 243-247.
[10] Annalakshmi. M., Chandramouleeswaran. M, An L- Fuzzy α− Supratopological TM-Subsystem, *International Journal of Computer Science*, ISSN:2348-6600.
[11] Annalakshmi. M., Chandramouleeswaran. M, An L - Fuzzy Connected on L - Fuzzy Topological TM-system, *International Journal of Mathematical Archive*- Vol-5, No.5, May 2018 pp.1-6.
[12] Annalakshmi. M, An L - Fuzzy α− supracontiuous in α− supratopological TM-System, *International Journal of Mathematical Archieve*-10(11), 2019, ISSN 2229-5046, PP 1- 6.

[13] Annalakshmi. M, An L - Fuzzy supratopological TM-System, *International Journal of Computer Science*, Jan 2018 ISSN 2348-6600 PP 410- 416.
[14] Annalakshmi. M, L-Fuzzy connected on L-Fuzzy Supratopological TM-Systems, Recent advances in Arts, Science and social sciences, ISBN:978-81-951746-3-8.
[15] Annalakshmi. M, An Intuitionistic L-Fuzzy topological TM-systems, Journal of Information and Computational Science - ISSN 1548-7741, vol.12,Issue.1-2022, pg 26-33.
[16] Annalakshmi. M, An Intuitionistic Fuzzy Topological TM-Subsystems, International *Journal of Research and Analytical Reviews*, Vol 9, Issue 1,660-666.
[17] Annalakshmi. M, An Intuitionistic L-Fuzzy Topological TM-Subsystems, International *Journal of All Research Education and Scientific Methods*, Vol 10, Issue 2.
[18] Annalakshmi.M, AL-Structure on FuzzyRelation Supratopological Systems, *International Journal of Research And Analytical Reviews*, Vol 10, Issue 4, pp. 485-491.
[19] Annalakshmi. M., N.Sugantha Meena. AL-Structure on F- Relation L-Fuzzy Topological Systems, *International Journal of All Research Education and Scientific Methods*, Vol 12, Issue3, pp. 3202-3206.
[20] Chang C.L., Fuzzy Topological Spaces, *Journal of Mathematical Analysis and Applications*, Vol.24, (1968), pp. 182-190.
[21] Gyanendrakumar panda, On the theory of fuzzy relations, *Technical research organisation*, India, ISSN: 2395-7786, vol 2, Issue 4.
[22] Imai.Y and Iseki.K., On axiom systems of Propositional Calculi , XVI Proc. *Japan Acad. Ser A. Math. Sci*., 42 (1966), 19-22.
[23] Iseki. K. On BCI-algebras, Math. *Seminar Notes*. 11(1980), pp.313-320.
[24] Lowen. R, Fuzzy topological spaces fuzzy compactness, *J.Math.Anal.Appl*.56 (1976), 621-633.
[25] Tamilarasi. A and Megalai. K , TM− algebra an introduction, CASCT., (2010).
[26] Wong.C.K ,Fuzzy Topology: Product and quotient theorems, *J.Math. Anal.Appl.* 45(1974), 512-521.
[27] Zadeh.L.A, Fuzzy sets, Inform.control 8 (1965),338-353

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